Bargaining Strength and the Formation of Coalition Governments

Josh Cutler, Duke University
Scott de Marchi, Duke University
Max Gallop, Duke University
Florian Hollenbach, Duke University
Michael Laver, New York University
Matthias Orlowski, Humboldt-University Berlin

Bargaining theory has been centrally important in game theory since the advent of the field (Nash, 1950). In large part, this is due to the range of phenomena bargaining theory applies to. In the legislative setting, bargaining is present from the birth to the conclusion of governments. Of particular importance in parliamentary systems is the creation of ruling coalitions; in most cases, a simple majority is not achieved after an election and parties must bargain over membership and the perquisites of belonging to a coalition.

The problem of government formation has also achieved significance as the primary empirical test for the main implications of non-cooperative models of bargaining. It is nearly ideal in many respects: election results provide a highly visible signal about the power of the agents involved in bargaining (i.e., the number of seats each party achieves in the parliament),[1] government membership is straight-forward and cabinet seats serve as a measure of perquisites, and agents are thought of as highly motivated and informed given the large stakes involved. In short, we have an election that determines the power of agents, a bargaining game occurs, and the stakes (cabinet seats) are divided between the members of the resultant coalition. On a theoretical level, the extension of Rubinstein bargaining (1982) by Baron and Ferejohn (1989) to include multiple players and an infinite horizon suggests a number of empirical tests based on this model of government formation.

There is, however, a conceptual problem that threatens to derail the entire endeavor. As Snyder, Ting, and Ansolabehere (2005) and Ansolabehere, Snyder, Strauss, and Ting (2005) (hereafter, ASST) point out, it is unresolved how one would apply the Baron-Ferejohn framework to the important class of non-homogenous bargaining games.[2] Non-homogenous games are those in which the minimum winning coalitions (MWC's) have different strengths; for example, the five party case of {4,3,3,2,2} has MWC's with strengths 8, 9, and 10. While this seems to be a minor technical matter, it has actual significance. One cannot determine equilibrium outcomes without solving this problem.[3]

Given this problem, ASST's plan to test Baron-Ferejohn bargaining games is elegant. The first step rests upon a formal proof: ASST attempt to prove that for both homogenous *and* (crucially) non-homogenous games:

> "the non-cooperative bargaining model of Baron and Ferejohn (1989) leads naturally to the result that expected payoffs are proportional to [minimum integer] voting weights" (p. 1, Snyder, Ting, and Ansolabehere, 2005).

For this reason, Snyder, Ting, and Ansolabehere argue that researchers cannot rely on raw voting weights and should instead use minimum integer weights (MIW's) which more

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[1] By assumption, parties are assumed to be unitary actors.

[2] A non-homogenous game is one in which not all minimum winning coalitions (MWC's) in a MIW representation have the same aggregate weight.

[3] Equilibrium payoffs are referred to as continuation values in the bargaining literature. Generally, they serve as a guide to what we would expect parties to come away with after bargaining.

accurately reflect the ability of parties to form winning coalitions.[4] For example, in the three party case where raw weights are {10,10,1}, each party has equal ability to form winning coalitions and thus has equal strength in the MIW representation of {1,1,1}.

If their deductive framework is correct and MIW's are proportional to the equilibrium outcomes expected for each party, then the second step is to construct an empirical test relating MIW's to perquisites. If one can show that MIW's predict perquisites, then Baron-Ferejohn style bargaining has empirical validity. Accordingly, ASST use data from parliamentary systems between 1946 and 2001 and a simple linear model to test for this relationship.

It is important to distinguish between minimum integer and raw seat shares to measure the strength of different parties because ASST believe that MIW's (not raw weights) are proportional to equilibrium values:

> "Empirical studies of coalition formation that use shares of seats to measure bargaining strength suffer from measurement error. The measurement error will be both random and systematic, as the correspondence between seats and weights is not one-to-one and not linear. Standard regression analyses will therefore produce biased estimates of the bargaining advantage that parties gain solely from their votes". Of particular concern, Browne and Franklin (1973), Browne and Frendreis (1980), Warwick and Druckman (2001), and others use seat shares as a proxy for shares of voting weights to predict the division of cabinet posts. The coefficient on seat shares will tend to be biased toward zero as a result of measurement error. Warwick and Druckman (2001) also include a dummy variable for the party chosen to serve as *formateur*. They find that the formateur effect is small and not different from zero, but note that measurement error in the voting weights might bias the estimated *formateur* effect" (Ansolabehere, et. al., p. 9-10).

The prior example proves their point. Imagine three parties with raw weights in the parliament of {10,10,1}. The MIW's for these same parties are {1,1,1}. Obviously, choosing to use the former as an independent variable to measure "bargaining power" will produce different estimates than the latter. Moreover, ASST assert that *only* MIW's are proportional to equilibrium outcomes (both certainly cannot be). While it makes good sense to use MIW's – after all, each party in this example, despite very different raw weights, is equally pivotal in forming coalitions – MIW's are often very difficult to calculate once one gets past simple cases. Parties in parliamentary democracies are highly motivated elites, but it is not obvious they can intuit the algorithm behind MIW's, especially as the number of parties grows large.[5]

ASST illuminate how thorny bargaining becomes when one considers the tandem problems of non-homogenous bargaining contexts and whether parties use MIW's or raw weights. In actual

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[4] MIW's are a vector of bargaining weights for the parties that are the smallest set of integers that generate the same set of winning coalitions as do the raw vote shares.

[5] The algorithm is used by most empirical researchers in political science and economics and is due to Aaron Strauss (2003), who was awarded a master's degree in computer science at MIT for the work. By and large, the code returns the correct answer; but, given certain sequences of problems to calculate, it ends up returning errors and does not stop doing so until the code is restarted. This means that for most users, the code is accurate most of the time, but in the case of doing large numbers of observations (say, for a dataset), there are problems.

PR systems since 1945, over 1/3 of all elections are non-homogenous.[6] And, as even the simple example above shows, MIW's and raw weights are very different creatures and any applied statistical model would need to choose which measure of power best captures party behavior.

Finally, it is worth noting that ASST also test the other main implication of Baron-Ferejohn bargaining models: the presence of a *formateur* advantage. In sequential bargaining games, the *formateur* is the party that is chosen by an exogenous selection mechanism to propose the first potential coalition.[7] In equilibrium, this confers an advantage to the *formateur* and one can test for this directly if one can ascertain who the *formateur* is in PR systems. Thus, ASST have identified two key variables – MIW's and *formateur* status – and argue that these are sufficient to construct a dispositive empirical test.

In many respects, we laud the work by Ansolabehere, Snyder, Strauss, and Ting – formally, they attempted to expand the class of phenomena covered by the Baron-Ferejohn model. Empirically, they tested two of the main implications of non-cooperative game theory against real-world data, following in the tradition of Browne and Franklin (1973), Morelli 1999, Fréchette et al. 2005, Warwick and Druckman (2006), and Golder, Golder, and Siegel (2012).

The main issue, which genuinely bedevils all work in this area, is the subset of observations that are non-homogenous. ASST's work in this area was noteworthy for identifying the problem, but unfortunately, their formal work bridging the gap between homogenous and non-homogenous games is incorrect (see Laver, de Marchi, and Mutlu, 2011). What this means is that direct empirical tests of non-cooperative bargaining models in the Rubinstein / Baron-Ferejohn tradition are impossible.

Substantively, this means we will focus in this paper on a more modest goal: the question of how parties assess bargaining power. Ultimately, it is of great interest what sort of model parties use when they bargain, but it is also useful to determine how parties assess power in the first place. Given the important empirical work of Warwick and Druckman which calls into question whether parties use MIW's or raw weights in determining power, it is worthwhile to sort out whether they or ASST are correct.

There are also two sets of methodological problems which bedevil the empirical work to date which deserve to be clarified. The first set is specific to ASST'S work and the second to all previous empirical work in this area. Thus, while we share the goal of prior researchers in this area – predicting the membership of coalition governments and assessing which parts of non-cooperative bargaining theory are empirically supported – we must first establish the correct principles for building a statistical model to test different notions of bargaining power. The more general problem of testing a specific bargaining model must wait on future work.

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[6] From ASST's replication data.

[7] This additional power is above and beyond that normally expected for the party given its share of seats in the parliament.

Problems with ASST

The first problem is that ASST's test of the *formateur* effect is not replicable due to the fact that it is impossible to code the variable. As noted in Laver, de Marchi, and Mutlu (2011),

> "the grounding institutional assumption of alternating offers bargaining models is an exogenous automaton that first selects a *formateur* and then reveals this as common knowledge to all agents. We rarely observe this revelation in the real world, but empirical analyses of BF-style bargaining over government formation fundamentally require coding "*formateur* status" of each political party, observed at the start of the bargaining process. The codings of *formateur* status that underpin STA's empirical work were supplied by Warwick (Ansolabehere et al., 2005: 556). Consulting Warwick and Druckman (2001: 634), we see that *formateur* status was coded from *Keesing's Contemporary Archives*. The following entry in *Keesing's* describes the formation of a German government in 2005. Crucially, this deals with events leading up to, but not including, the eventual formation of a government. It is thus a description of legislative bargaining, taken from the primary source in this field, but one that *does not use the benefit of hindsight* about the eventual outcome of the process under analysis:
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> 'After the results were declared, Schröder controversially claimed that he was the victor because the SPD remained the largest single party, discounting the fact that the CDU and the CSU formed a single group in the Bundestag. Merkel responded that, as the leader of the largest parliamentary group, she had the right to head a new government. However, talks between her and the Greens on Sept. 23 on the formation of a "Jamaica" majority coalition – named after the black (CDU/CSU), yellow (FDP), and green colors of the Jamaican flag – quickly failed. At the same time, the FDP maintained its refusal to enter a "traffic light" coalition with the SPD and the Greens. The only viable option for a majority government, therefore, was a "grand coalition" of the CDU/CSU and the SPD, although at end-September Merkel and Schröder were both still insisting that they should be Chancellor.'
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> Who, on this basis, should be coded as exogenously determined common knowledge *formateur*?" (Laver, de Marchi, and Mutlu, p. 5).

More seriously, they also find that the ASST coding for *formateur* is nearly isomorphic with the party that eventually takes the prime ministership. Obviously, this calls into question whether the coding is exogenous or endogenous and it completely ignores the strategic considerations in choosing a prime minster (Gasgow, Golder, and Golder, 2011).

Warwick and Druckman (2006) identify a second problem with ASST's empirical findings: they challenge ASST's use of MIW's instead of raw seat shares. They believe that raw seats are better predictors of coalition membership and that the *formateur* effect vanishes once one simultaneously tests raw seats. Obviously, this is a serious criticism given ASST's strong belief that MIW's better capture the reality of bargaining power. In addition, Warwick and Druckman find that *formateur* status is correlated with large parties, further undermining ASST's statistical model.

Given the above, our approach here is to drop the *formateur* variable. It is difficult to justify its use: coding the variable has been poorly done and even when it is included its performance is poor. We do, however, retain MIW's and discuss this more fully below. Our core idea is that while Baron-Ferejohn models are not supported by current empirical work, it is still of vital importance whether parties use accurate assessments of bargaining power (i.e., MIW's) or rely on cognitively less demanding assessments (i.e., raw seat shares).

The third problem is identified by ASST themselves and concerns the uniqueness of MIW's. As they note, "homogenous games have a unique minimal integer representation", but non-homogenous games do not:

> A minimum integer representation of a voting game is unique whenever there are five or fewer parties because in these situations all minimal winning coalitions share the same total weight (which makes the game 'homogeneous'); in larger games, however, the integer representation may not be unique… For some non-homogeneous games, the formulation below will only approximate the true relationship (ASST, p. 6).

Warwick and Druckman repeat this warning and given the large proportion of games in real world systems that are non-homogenous, we are left with a sense of foreboding. For obvious reasons, we would like our key independent variable to be uniquely measured. How serious is this problem?

The best treatment of the problem is found in Strauss (2003) where he uses the following example to illustrate the non-uniqueness and non-monotonicity of MIW's:

	Parties												
	1	2	3	4	5	6	7	8	9	10	11	12	Total
Original	56	53	47	46	37	36	30	10	6	6	4	3	334
Minimum Integer	55	52	46	45	36	35	30	10	6	6	3	4	328
Constrained Integer	61	58	51	50	40	39	34	11	7	7	4	4	366

(Strauss, p. 31)

The main issue with MIW's is whether or not equivalent parties should have equal weights and to a large degree, stating that they are not unique or monotonic is only because the definition of what constitutes a MIW is not settled. The central idea at stake is "equivalence", which is defined as two parties that are perfect substitutes. The original vector of raw weights is the first row in the forgoing table; the second row is the MIW representation without assuming that equivalent parties have equal weights; the third row is the MIW representation assuming equivalency.

Following Freixas and Kurz (2011), we replace MIW's with minimum integer weights preserving types; i.e., parties that are perfect substitutes have equal weights.[8] As Freixas and Kurz note, this problem is especially acute with non-homogenous games and can occur with as few as eight parties. The advantage with using MIW's preserving types is that with this definition weights are unique and monotonic. The loss is that in some rare cases as Strauss points out

[8] We will, however, continue to refer to these weights as MIW's in what follows. The Freixax and Kurz definition should, in our opinion, be universally adopted.

above, the resultant vector will be greater than the original vector of raw weights. To the extent that we want to test an accurate measure of bargaining power empirically, this tradeoff seems more than worthwhile; in all cases, we would like equivalent parties to be treated as such in our measure of bargaining power.

The last problem with ASST's work is that the algorithm they employ for their MIW calculator is flawed. Most often, it provides the correct solution to a vector of raw party weights; but, depending on inputs, the calculator will produce incorrect outputs and continue to do so – this is particularly problematic if empirical researchers use the calculator to fill in values for MIW's for an entire dataset.

For the analyses contained here, we have written our own MIW algorithm and have verified that the code is correct in all cases (though, see below on the definition we adopt for MIW's). As with any algorithm for calculating MIW's, we must first produce all minimum winning coalitions; we do this recursively:

```
#define recursive coalition combinatorics generator
#note this finds ALL coalitions, not just MWC's or min. weight coal.'s
def combo(tcoal, tlim, n, L1, tl):
    # tcoal is coalition list, tlim is # of parties in coal, n is starting party
    # and L1 is list of coalitions used to fill dictionary coal_list
    for j in range(n,T-tlim+1):
        tl.append(j)
        L1.append(tcoal[j])
        if tlim-1>0:
            combo(tcoal, tlim-1, j+1, L1, tl)
        if tlim==1:
            tsum=0
            for z in range(0,len(L1)): tsum=tsum+L1[z]
            coal_list[tuple(L1)]=tsum
            if E%2==0 and tsum==(u-1):
                tie_list.append(tuple(tl))
            elif tsum>=u:
                pulp_list.append(tuple(tl))
                for k in L1:
                    if tsum - k >= u:
                        pulp_list.pop()
                        break
            else:
                loss_list.append(tuple(tl))
        L1.pop()
        tl.pop()
    return 0
```

Solving for MIW's is a straight-forward application of linear programming and we use the PuLP library to implement our algorithm (http://packages.python.org/PuLP/).[9]

[9] For the best verbal description of the algorithm, see Strauss (2003). His code is not, however, open-source, though the java program can be run from http://www.mindlessphilosopher.net/weights/. Our version of the

Problems Generally with Empirical Tests

In addition to the problems specific to ASST's work in this area, there are also a set of five general problems that apply to all empirical work in this area. All researchers to date have relied on data from European democracies in the post-war period and have used largely the same dataset (see below for more details).[10]

First, the data are not IID, insofar as there are two distinct types of government formation, inter-electoral and post-electoral. Inter-electoral government formations are triggered by a failed equilibrium and are not modeled in this literature – cabinet reshuffles and new coalitions are not caused by electoral changes in the power of the respective parties. Within the post-electoral cases there are two distinct types of elections: governments may collapse (e.g., due to a vote of no-confidence), or elections may occur because of constitutional provision (e.g., in Britain, a new election must be called within five years of the last election). Datasets used by researchers in this area unfortunately mingle all of the above cases, without distinction. Minimally, one would want to include indicator variables to separate out the different cases. Maximally, one could easily argue that the DGP's behind these different events are distinct enough that one should not lump them all together.

Second, not all observations in the data have equal weight. Different nations have both unequal numbers of parties and unequal frequency of elections; without care, models that ignore this problem have much more to do with Italy than they should. Italy, notably, has a large number of political parties as well as frequent elections. In our data, we have sixteen nations, but Italy accounts for over 18% of the total observations. Italy, Belgium, Denmark, and Finland together account for half of all observations.

Third, the dependent variable – the number of cabinet seats – is not normally distributed. While we present the results of OLS regressions below, we supplement these with MLE models based on a mixed continuous–discrete distribution. The dependent variable has a limited range (since it is a proportion). To some degree, this suggests the choice of a beta distribution, which is flexible enough to suit the problem (see Brehm and Gates, 1993 on this topic). But, beta distributions (because they are continuous) do not assign positive probability to any particular value in the range of the dependent variable. As a cursory inspection of the dependent variable in this case reveals, there are a large number of 0's, which argues for a mixed distribution – in taking this approach, we follow the work of Ospina and Ferrari (2012).

Fourth, all of the work in this area hinges on the interpretation of the significance of "theoretically relevant" variables. For example, ASST's work focused on the p-value attached to

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algorithm chooses a slightly different set of assumptions than Strauss (see p. 6 of this paper), is not prone to mistakes, but is slower for very large numbers of parties.

[10] Warwick and Druckman extend ASST's dataset by adding subject matter expert assessments of salience of portfolios; we have not weighted cabinet seats.

the *formateur* variable. In general, we dislike the overemphasis on p-values especially given the reliance on one sample which has been extensively studied. To the extent that particular independent variables are of theoretical interest, a validation strategy should be presented so that confidence in a particular variable is warranted.

Last, in the existing datasets, there are a relatively large number of missing or incorrect values. We correct these problems in our data.[11]

Data

The majority of our data comes from the Parliament and Government Composition Database (Parlgov). Parlgov was constructed by Holger Döring, Philipp Manow and collaborators and contains the election results and government formation data for all EU members as well as many OECD countries from 1945 forward (Parlgov 2012). Our dataset includes data from 1945 until the most recent available data on cabinet seats (generally the current government) for 16 parliamentary democracies in Western Europe.[12] We only include cabinets following an election and exclude inter-election observations.

We rely on Parlgov for data on elections, seat share, as well as the party of the prime minister. However, Parlgov did not include the number of cabinet seats per party in the governing coalition and instead included an indicator variable for coalition members. In addition we used Parlgov's coding for when governments dissolved and new governments were established.

To complement the Parlgov data set with our dependent variable we collected data on the number of cabinet seats per party from an online database.[13] Using the parliamentary parties from the Parlgov dataset we calculated and added the number of cabinet seats for each observation. We then also calculated the share of cabinet seats for each coalition.

As explained above, our main variable of interest is the MIW of each party's seat share in the parliament. Using the MIW calculator described in the previous section, we created MIW weights based on the Parlgov data on parliamentary seat share by party and added this to the existing dataset.

Results

We excluded all single party and minority governments from the data set since our theory does not apply here. Thus, our dependent variable, cabinet seat shares, ranges from 0 to less than 1, with a point mass at 0. For our first set of results, we model these data as a mixture between a

[11] In ASST's replication data, for example, Germany 's 1990 election is duplicated, the elections of 1953 and 1957 are missing, and there are "extra" elections in the 1960's that are likely only minor cabinet reshuffles.
[12] These countries are: Austria, Australia, Belgium, Denmark, Finland, Germany, Great Britain, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden.
[13] The data on the number of cabinet seats was collected from http://www.kolumbus.fi/taglarsson/dokumentit/governm2.htm.

beta distribution and a degenerate distribution in 0. Hence, the likelihood function we maximize is (Ospina and Ferrari 2012, p. 1611):

$$L(y;\nu,\mu,\sigma) = \prod_{i=1}^{n} {\nu_i}^{1-y_i}(1-\nu_i)^{y_i} \prod_{i:y_i\in(0,1)}(1-\nu_i)\frac{\Gamma(\sigma_i)}{\Gamma(\mu_i\sigma_i)\Gamma((1-\mu_i)\sigma_i)} {y_i}^{\mu_i\sigma_i-1}(1-y_i)^{(1-\mu_i)\sigma_i-1},$$

where:

$$\nu_i = \log\left(\frac{\beta_{\nu 0}+\beta_{\nu 1}\cdot miw\ share_i+\beta_{\nu 2}\cdot seat\ share_i}{1-(\beta_{\nu 0}+\beta_{\nu 1}\cdot miw\ share_i+\beta_{\nu 2}\cdot seat\ share_i)}\right),$$
$$\mu_i = \log\left(\frac{\beta_{\mu 0}+\beta_{\mu 1}\cdot miw\ share_i+\beta_{\mu 2}\cdot seat\ share_i}{1-(\beta_{\mu 0}+\beta_{\mu 1}\cdot miw\ share_i+\beta_{\mu 2}\cdot seat\ share_i)}\right),$$
$$\sigma_i = \log(\beta_{\sigma 0}+\beta_{\sigma 1}\cdot miw\ share_i+\beta_{\sigma 2}\cdot\ seat\ share_i)$$

Link functions were chosen to constrain parameters to the unit interval (in the case of ν_i and μ_i) and to be strictly positive in the case of the precision parameter σ_i. The parameter ν_i represents the likelihood of a zero observation, while μ_i is the mean for the beta distribution and σ_i is the precision of the beta distribution.

In order to account for the different weights of parties in the estimation due to the differences in party system sizes and frequency of elections, we calculated a probability for each party to be selected into a subsample of our data. This selection probability is inversely related to the number of parties in parliament. It is calculated as:

$$\pi_i = \frac{1}{n_k}\cdot\frac{1}{K}$$

Where n_k is the number of parties in a given cabinet k and K is the total number of cabinets in the truncated data set.[14] Based on the selection probabilities, 80% of the observations (1134 parties) in the truncated data set are randomly selected in a subsample. In a next step, the subsample is divided into training and test set by randomly selecting 80% of the observations in the subsample with a uniform probability.

We then fitted the model described above on the training set for those cabinets where the total number of parties with non-zero minimum integer weights is less or equal to 8, and those cabinets where there are more than 8 parties with non-zero minimum integer weights.[15] For the three subsamples, we predicted the cabinet seat shares of the parties in the test set based on the model estimates and calculate the root mean squared error. To account for the unequal

[14] There are more aggressive strategies we could have used in reweighting the data. The approach followed here is much like using the bootstrap – we are seeing how fragile the results are without making any huge changes to the sample.

[15] We chose the threshold of 8 because it was a natural inflection point in the amount of time our algorithm took to calculate MIW's. For robustness, we also examined models using 7 or 9 and the results were similar (though as one exceeds 8 the number of observations in the "high" complexity group falls off sharply).

selection probabilities, we draw 1000 subsamples from the whole data set, split them into training and test sets and fitted the model.

The distribution of estimation results are reported in Table 1 to Table 3 below. What is obvious is that both MIW's and raw weights affect the number of cabinet seats parties receive as the outcome of bargaining. The two parameters of most interest are ν (which shows the relationship between the independent variables and having no cabinet seats) and μ (which shows the relationship between the independent variables and a strictly positive proportion of cabinet seats). Interestingly, MIW's are strongly correlated with inclusion in the coalition (i.e., a negative relationship with 0 values for the dependent variable) but as μ shows in Table 1, raw weights outperform them when it comes to the number of seats a party receives once it is in the coalition.

There is also a strong effect for complexity: if the number of non-zero parties is greater than eight, raw weights do a relatively better job predicting outcomes than in the case where there are fewer parties. If one looks at the values for ν (and to a somewhat lesser degree with μ) between Tables 2 and 3, it is obvious that MIW's do much better in the low complexity subsample. Given the complexity of the algorithm for calculating MIW's, this is at one level is not surprising, but it does represent a novel finding that the use of a model by elite agents is still constrained by the complexity of the problem (i.e., as the key parameter of interest -- number of parties -- increases).

For comparison to prior results, we also present OLS regressions (despite the fact that they are misspecified). Largely, they are in accord with the zero inflated beta models presented above, but a few additional details are worth pointing out. First, models were trained on eighty percent of the data, and it is worth noting that they performed equally well out-of-sample. The MSE in sample was .0275 versus .0277 out-of-sample for the model presented in Table 4; additional support for the role played by the two independent variables of interest (i.e., MIW's and raw weights) is provided by the bootstrapped estimates in Table 5. Second, Table 4 demonstrates that MIW's are in fact utilized by parties. Table 6, interestingly, shows that MIW's are far less predictive of cabinet seat allocations once one constrains the sample to only those parties that are in the government. In qualitative terms, this means that MIW's predict entry to coalitions, especially in the low complexity case of eight or fewer effective parties. But, raw weights are dominant in predicting seat shares once the coalition is established, as demonstrated in Table 6.

Discussion

We have demonstrated that MIW's are in fact important in explaining coalition outcomes in parliamentary democracies. Put another way, political parties have very sophisticated assessments of relative bargaining power when they form coalitions, especially when it comes to inclusion or exclusion from the ruling coalition. When it comes to allocating cabinet seats to members of the ruling coalition, however, parties rely more heaving upon raw weights for reasons that are worth further study. Finally, we find that the cognitive ability of parties to assess bargaining power is not unlimited. When the complexity of the problem exceeds a computational threshold, parties rely upon the more naïve measure of bargaining power.

Table 1: Distribution of 1000 parameter estimates and root mean squared error for out of sample predictions for all cabinets

		Mean	Std. Dev.
ν	Intercept	1.103	0.053
	MIW	-3.392	0.547
	Raw Weights	-2.239	0.424
μ	Intercept	-2.026	0.029
	MIW	0.668	0.155
	Raw Weights	5.728	0.201
σ	Intercept	3.384	0.117
	MIW	-0.456	0.764
	Raw Weights	-2.136	0.918
	RMSE	0.177	0.005

Table 2: Distribution of 1000 parameter estimates and root mean squared error for out-of-sample predictions for cabinets with at most 8 parties with non-zero minimum integer weights in parliament

		Mean	Std. Dev.
ν	Intercept	1.066	0.095
	MIW	-5.079	0.764
	Raw Weights	-0.223	0.572
μ	Intercept	-1.827	0.056
	MIW	1.030	0.468
	Raw Weights	4.780	0.495
σ	Intercept	2.538	0.183
	MIW	3.075	2.676
	Raw Weights	-3.184	2.462
	RMSE	0.215	0.008

Table 3: Distribution of 1000 parameter estimates and root mean squared error for out-of-sample predictions for cabinets with more than 8 parties with non-zero minimum integer weights in parliament

		Mean	Std. Dev.
ν	Intercept	1.180	0.065
	MIW	-0.276	0.983
	Raw Weights	-6.319	0.834
μ	Intercept	-2.186	0.025
	MIW	-0.163	0.197
	Raw Weights	7.041	0.208
σ	Intercept	3.886	0.080
	MIW	-4.518	0.936

Raw Weights	1.542	0.832
RMSE	0.140	0.004

Table 4: OLS regression dropping cabinet shuffles (i.e., no election) Training Set

Source	SS	df	MS
Model	64.5252976	4	16.1313244
Residual	41.8305317	1517	.02757451
Total	106.355829	1521	.069924937

Number of obs =	1522
F(4, 1517) =	585.01
Prob > F =	0.0000
R-squared =	0.6067
Adj R-squared =	0.6057
Root MSE =	.16606

cabinet_se~e	Coef.	Std. Err.	t	P>|t|	Beta
miw_share	.7275281	.0383605	18.97	0.000	.4894914
seats_share	.0059366	.0004554	13.04	0.000	.3374586
italy	.0215915	.0147621	1.46	0.144	.0247732
bicameral	.0041572	.0090364	0.46	0.646	.007711
_cons	-.0505036	.0078699	-6.42	0.000	.

Table 5: Bootstrapped results from Table 4 Full Sample

Linear regression

Number of obs =	1900
Replications =	1000
Wald chi2(4) =	2370.66
Prob > chi2 =	0.0000
R-squared =	0.6092
Adj R-squared =	0.6083
Root MSE =	0.1661

cabinet_se~e	Observed Coef.	Bootstrap Std. Err.	z	P>|z|	Normal-based [95% Conf.	Interval]
miw_share	.7168528	.0368828	19.44	0.000	.6445639	.7891418
seats_share	.0058645	.0006005	9.77	0.000	.0046875	.0070416
italy	.0168754	.0108424	1.56	0.120	-.0043754	.0381261
bicameral	.0012766	.0083916	0.15	0.879	-.0151707	.0177239
_cons	-.0475859	.0063527	-7.49	0.000	-.060037	-.0351348

Table 6: OLS Regression excluding non-cabinet members

Source	SS	df	MS
Model	34.3393443	4	8.58483607
Residual	9.34253354	474	.019709986
Total	43.6818778	478	.091384682

Number of obs =	479
F(4, 474) =	435.56
Prob > F =	0.0000
R-squared =	0.7861
Adj R-squared =	0.7843
Root MSE =	.14039

cabinet_se~e	Coef.	Std. Err.	t	P>|t|	Beta
miw_share	.161335	.0452712	3.56	0.000	.1301712
seats_share	.0141865	.000658	21.56	0.000	.7858052
italy	.0381817	.0245511	1.56	0.121	.0349783
bicameral	-.0337654	.0136232	-2.48	0.014	-.0549325
_cons	.065454	.0136862	4.78	0.000	.

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